EULERIAN MAGIC WORD SQUARES

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In the May 1933 Word Ways, Lee Sallows introduced a form of gematria he called *wints* (word integers), based on the use of the alphabet and a space to express numbers in base 27. (In normal gematria, A=1, B=2, etc.) His article included 3x3 and 4x4 magic squares made from wints and found by computer search. In May 1996 he presented additional 3x3 squares found by computer search. In the original article, many of the magic constants were wints. This was not true for the later set. Most are simply Euler squares which become magic for any gematria.

An Euler square, also known as a Graeco-Latin square, is an important tool of statisticians and mathematicians. As illustrated in Figure 2, it consists of an nxn array of two-digit numbers, each digit of which is 0,1,2 ... or (n-1). Each units digit appears once in every row and once in every column, as does each tens digit; furthermore, every possible combination of two digits is represented (00,01,02 ... (n-1)(n-1)). Eulerian squares exist for most values of n; 6 is a famous exception, proved, as Euler himself conjectured, to be impossible.

The tens digits need not be 0,1,2 ... ; they can be any set of distinct numbers such as 22,31,48,921,2882. The units digits can be similarly generalized, and need not match the tens digits; however they must all have the same number of digits. The construction of Euler word squares is based on this generalization.

I now show how to construct Euler word squares using a computer strictly as an ancillary device. I present two 5x5 squares for which the magic constant is a wint; I also present one 8x8 square. To start, I constructed the matrix of Figure 1. Starting with a somewhat larger array (which was later reduced) and using my word processor as a chalkboard, I shifted rows and columns to reveal the region of maximum density in the lower right. This region includes 8 rows and 7 columns. Using the letter-groups end, ill, ore, ins, ear, its, are, all to represent the units digits 0,1,2,3,4,5,6,7 and the letter-groups _,b,f,p,t,w,sh,sp to represent the tens digits 0,1,2,3,4,5,6,7, one converts the 8x8 Euler square in Figure 2 to the 8x8 Euler word square in Figure 3. (Since all 3-letter endings are themselves words, the _ assignment is possible.)

All words implied by Figure 1 are in Merriam-Webster's 10th Collegiate dictionary. Figure 1 allows us to extract many squares of various sizes but it took a full day of trial and error to find one which when taken as a wint square has a magic constant which is also a wint. The magic constant for Figure 4 can be found in Answers and Solutions.

In his initial article, Sallows asks for a magic square whose constant is the wint "magic". This required going to the unabridged dictionaries to allow making Figure 5. This square would not have been found by a computer search even if I had used a vocabulary many times as large as Sallows's.

I have been making arrays like Figure 1 for some time. Sets of beginning-ending combinations are useful for many puzzles. Figure 6 was in my notes. Figure 7 is constructed from it. Most of the 7-, 8- and 9-letter words come from unabridged dictionaries.

For ordinary gematria, return to Figure 1. These Euler magic squares are presented in condensed form:

(d,g,r,s,st) x (ags,ale,ate,ash,ill) = word ways journal
(d,g,h,m,r) x (ash,ate,ays,eed,ill) = ross eckler editor
(b,f,p,sh,sp) x (ear,ill,ine,ins,ore) = dave morice kickshaws editor
(b,h,r,s,w) x (ags,ail,ash,ays,ill) = leonard gordon author

	st	d	g	h	r	m	s	b	f	p	t	w	sh	sp
ail				x	x	x	x	x	x	x	x	x		
ang		x	x	x	x		x	x	x	x	x			
eed	x	x	x	x	x	x	x		x	x		x		x
ale	x	x	x	x	x	x	x	x		x	x	x	x	
ags	x	x	x	x	x	x	x	x	x		x	x	x	
ash	x	x	x	x	x	x	x	x	x	x		x		
ate	x	x	x	x	x	x	x	x	x	x				x
ays	x	x	x	x	x	x	x	x	x	x		x	x	x
ill	x	x	x	x	x	x	x	x	x	x	x	x	x	x
end					x	x	x	x	x	x	x	x	x	x
ore	x		x			x	x	x	x	x	x	x	x	x
ins		x	x	x			x	x	x	x	x	x	x	x
ear	x	x	x	x	x		x	x	x	x	x	x	x	x
its		x		x			x	x	x	x	x	x	x	x
are	x	x		x	x	x		x	x	x	x	x	x	x
all	x		x	x		x		x	x	x	x	x	x	x
ine	x	x				x	x	x	x	x	x	x	x	x

<-figure 1

34	41	03	10	22
13	20	32	44	01
42	04	11	23	30
21	33	40	02	14
00	12	24	31	43

12	03	30	21
31	20	13	02
23	32	01	10
00	11	22	33

figure 2.

17	50	43	04	32	75	66	21
31	76	65	22	14	53	40	07
00	47	54	13	25	62	71	36
26	61	72	35	03	44	57	10
45	02	11	56	60	27	34	73
63	24	37	70	46	01	12	55
52	15	06	41	77	30	23	64
74	33	20	67	51	16	05	42

56	61	03	15	20	32	44
35	40	52	64	06	11	23
14	26	31	43	55	60	02
63	05	10	22	34	46	51
42	54	66	01	13	25	30
21	33	45	50	62	04	16
00	12	24	36	41	53	65

ball	wend	tins	ear	pore	spits	share	fill
pill	spare	shits	fore	bear	wins	tend	all
end	tall	wear	bins	fits	shore	spend	pare
fare	shill	spore	pits	ins	tear	wall	bend
tits	ore	bill	ware	shend	fall	pear	spins
shins	fear	pall	spend	tare	ill	bore	wits
wore	bits	are	till	spall	pend	fins	shear
spear	pins	fend	share	will	bare	its	tore

<--figure 3.

tins	wend	ore	sail	fill
sore	fail	till	wins	end
will	ins	send	fore	tail
fend	tore	wail	ill	sins
ail	sill	fins	tend	wore

figure 4.

rave	leant	are	bads	cach
bare	cads	rach	leave	ant
leach	ave	bant	care	rads
cant	rare	leads	ach	bave
ads	bach	cave	rant	leare

figure 5

355622	6476834	1220	40222	59867
40586	59905	355112	6477035	1127
6476525	1328	40493	60269	355150
60176	355514	6476563	818	40694
856	40184	60377	355412	6476927

6933765 = magic

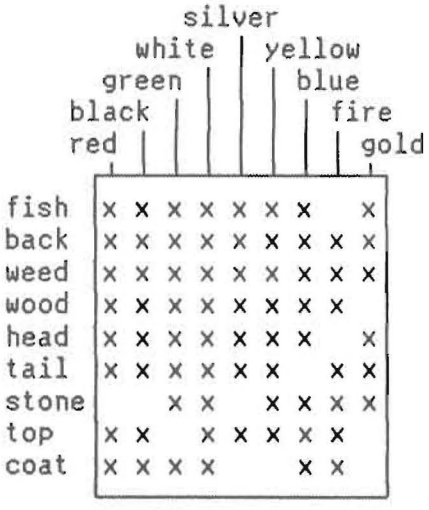

figure 6

blue-coat	black-back	fire-top	red-tail	white-wood
red-top	white-tail	blue-wood	black-coat	fire-back
black-wood	fire-coat	red-back	white-top	blue-tail
white-back	blue-top	black-tail	fire-wood	red-coat
fire-tail	red-back	white-coat	blue-back	black-top

figure 7